Objects $\rightarrow$ Properties / Prove theorems

$\downarrow$

more theorems, applications

Math = Fun

Calculus – Study of FUNctions

Calculus 3 –

Real Number Properties

$\forall a, b \in \mathbb{R}, \quad a+b = b+a$ ← Commutative Property of Addition.

"For all a, b in the set of Real #s, $a+b = b+a$."

$\forall a, b \in \mathbb{R}, \quad ab = ba$ ← Commutative Property of Multiplication

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$\forall a, b, c \in \mathbb{R}, \quad a+(b+c) = (a+b)+c$ ← Associative Property of addition.

the associative property for mult. is also true.

$\forall a,b,c \in \mathbb{R}$, ① $a(b+c) = ab + ac$ ② $(b+c)a = ba + ca$ } Distributive property

Example

Simplify: $3x^2(y + 4xy) - 2x^3y$

$= 3x^2y + 12x^3y - 2x^3y = 3x^2y + 10x^3y.$

~~Commutative property~~ of subtraction:

$\forall a,b \in \mathbb{R},\ a-b = b-a.$ False

True: $\forall a,b \in \mathbb{R},\ a-b = -(b-a)$

Multivariable Algebra

$\mathbb{R}^1 = \mathbb{R}$ = set of real #s.

$\mathbb{R}^2$ = set of ordered pairs of real #s.

$= \{(x,y) : x,y \in \mathbb{R}\}$

"the set of all pairs (x,y) such that x and y are in the set of real #s."

satisfying the condition that = where

$\mathbb{R}^3 = \{(x,y,z) : x,y,z \in \mathbb{R}\}$ = set of ordered triples of real #s.

$\mathbb{R}^4 = \{(x_1, x_2, x_3, x_4) : x_1, x_2, x_3, x_4 \in \mathbb{R}\}.$

$$\mathbb{R}^n = \text{set of } n\text{-tuples of real \#s}$$
$$= \{(x_1, x_2, x_3, x_4, \dots, x_n) : x_j \in \mathbb{R} \text{ for all } j \in \{1, \dots, n\}\}$$
$$= \{(x_1, x_2, \dots, x_n) : x_j \in \mathbb{R}\ \forall j\}.$$

Addition of elements of $\mathbb{R}^n$

sometimes called points
sometimes called vectors.

$$\left.\begin{aligned}&(x_1, x_2, \dots, x_n) + (y_1, y_2, \dots, y_n)\\ &= (x_1+y_1, x_2+y_2, \dots, x_n+y_n)\end{aligned}\right] \text{defn of addition in } \mathbb{R}^n.$$

Subtraction:

$$\left.\begin{aligned}&(x_1, x_2, \dots, x_n) - (y_1, y_2, \dots, y_n)\\ &= (x_1-y_1, \dots, x_n-y_n)\end{aligned}\right\} \text{Component-wise addition.}$$

$$x = (x_1, x_2, \dots, x_n)$$

components of the vector
entries of the vector.

Notation: In some books, they denote points with parentheses, and denotes vectors with $\langle \dots, \dots \rangle$

$$\left.\begin{aligned}&(1,2,3) \text{ point in } \mathbb{R}^3\\ &\langle 1,2,3\rangle \text{ vector in } \mathbb{R}^3.\end{aligned}\right\} \text{I will use (,) for both.}$$

$$1\hat{i} + 2\hat{j} + 3\hat{k} = (1,2,3) = \langle 1,2,3\rangle.$$

Scalar Multiplication

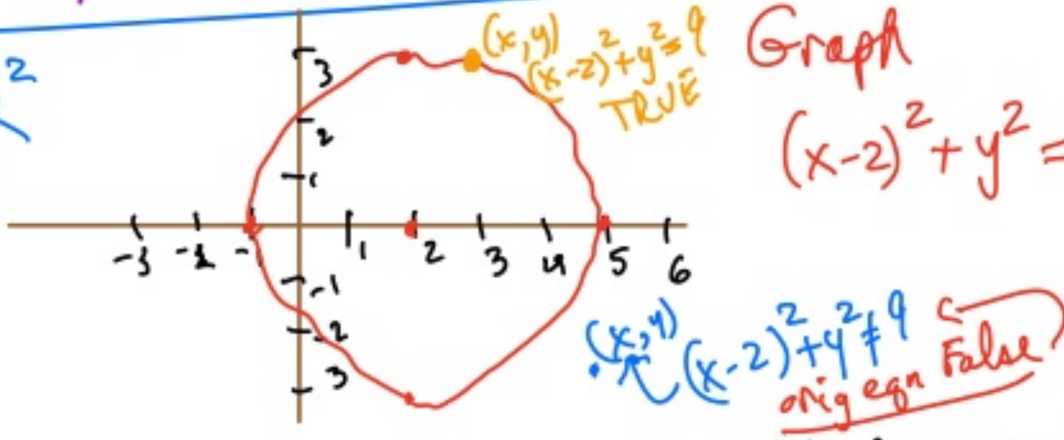
(scalar)(vector)

in $\mathbb{R}$ — in $\mathbb{R}^n$.

$$c\underbrace{(x_1, \dots, x_n)}_{\text{denoted } x \text{ or } \vec{x}} = (cx_1, \dots, cx_n)$$

cx or $c\vec{x}$.

$\mathbb{R}^2$

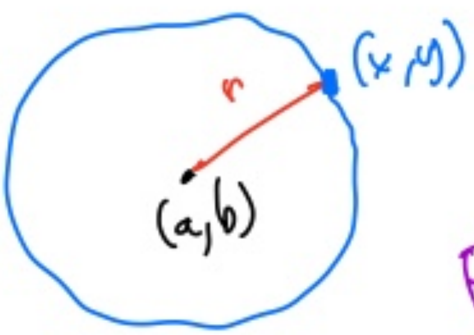


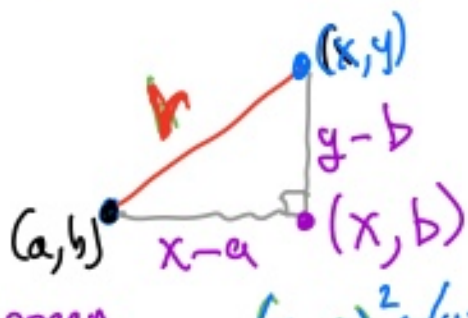
Graph $(x-2)^2+y^2=9$

When we say — graph of an equation in $\mathbb{R}^n$ — we mean the set of all points in $\mathbb{R}^n$ that makes the equation true.

Where does that equation of a circle come from?

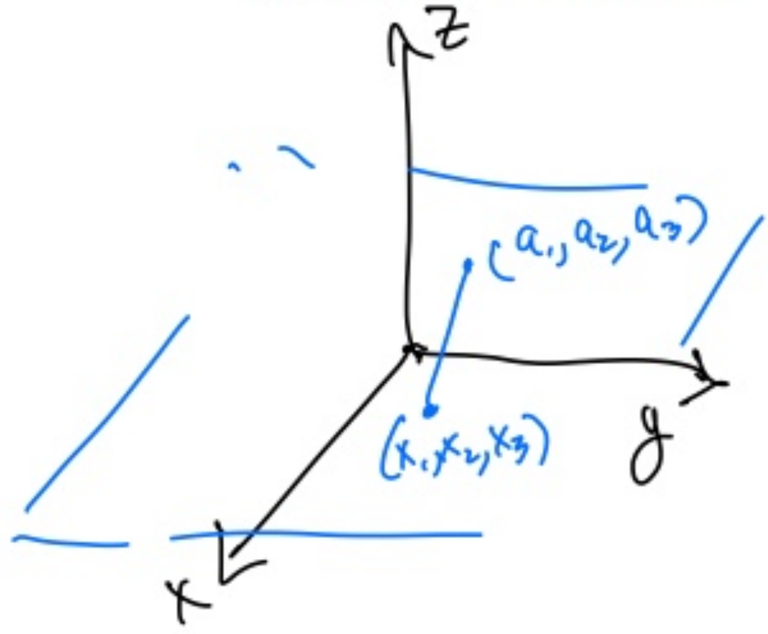
$$(x-a)^2+(y-b)^2=r^2$$

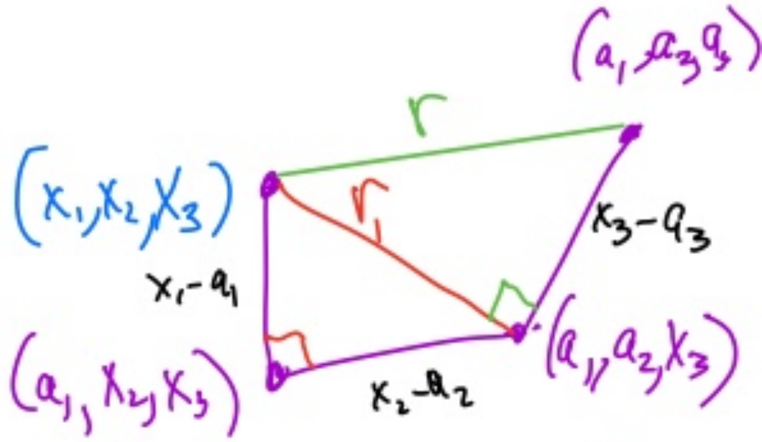




Pythagorean theorem $(x-a)^2+(y-b)^2=r^2$

Distance from (a,b) to $(x,y) = \sqrt{(x-a)^2+(y-b)^2}$





$$r_1 = \sqrt{(x_1-a_1)^2+(x_2-a_2)^2}$$

$$r = \sqrt{r_1^2+(x_3-a_3)^2}$$

$$r = \sqrt{\underbrace{(x_1-a_1)^2+(x_2-a_2)^2}_{r_1^2}+(x_3-a_3)^2}$$

$\Rightarrow$ Distance between (a_1, a_2, a_3) & (x_1, x_2, x_3)

$$= \sqrt{(x_1-a_1)^2+(x_2-a_2)^2+(x_3-a_3)^2}$$

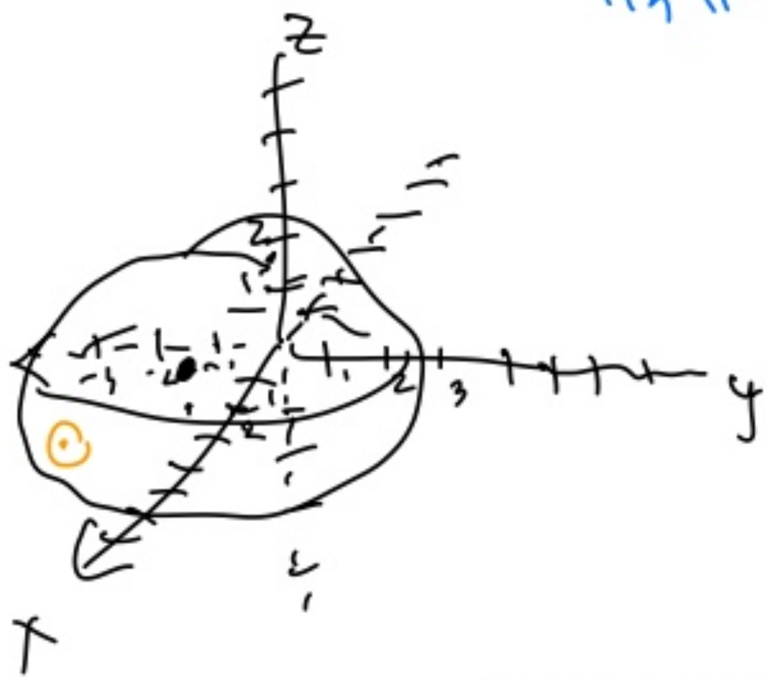
By induction, we could prove that the distance formula in $\mathbb{R}^n$ is

Distance from $(x_1, x_2, \dots, x_n)$ to $(a_1, a_2, \dots, a_n)$

$$= \sqrt{(x_1-a_1)^2+(x_2-a_2)^2+\dots+(x_n-a_n)^2}$$

$$\text{Distance} = \sqrt{\sum_{k=1}^{n} (x_k - a_k)^2}$$

Example: Graph $(x-2)^2 + (y+1)^2 + (z+1)^2 = 16$ in $\mathbb{R}^3$.



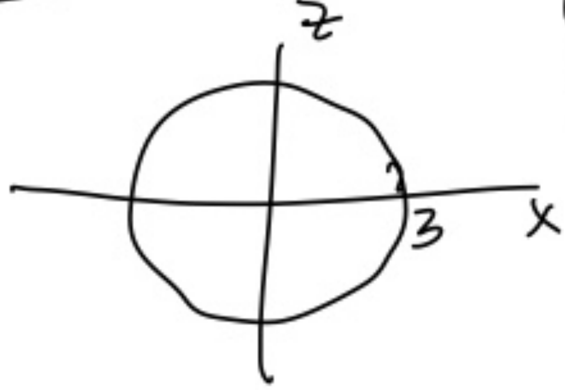
2-d sphere in $\mathbb{R}^2$

distance^2 from (x,y,z) to $(2,-1,-1) = 4$

sphere of radius 4 centered at $(2,-1,-1)$.

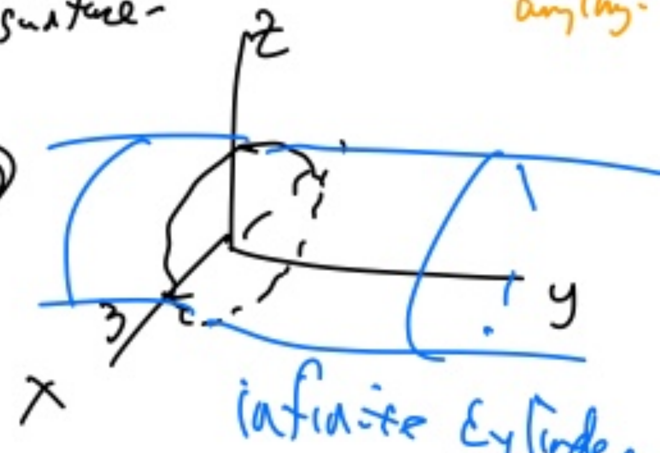
Example Graph $x^2 + z^2 = 9$ in $\mathbb{R}^3$

y can be anything.



$\underset{x}{(3},\underset{y}{0},\underset{z}{0)} \in$ surface.

$(3, -2.894, 0)$



infinite cylinder.